

A. D. Sankaranarayanan*, T.K. Sankaranarayanan

Department of Aerospace Engineering, I.I.T. Kanpur, UP 208016, India

Received 11 April 2005; accepted 20 October 2005; available online 25 October 2005

Abstract

In this paper, we study the evolution of a vortex structure in a 2D flow field. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS.

© 2005 Elsevier Inc. All rights reserved.

Keywords: Vortex; DNS; 2D; Vortex stretching; Vortex tilting

1. Introduction

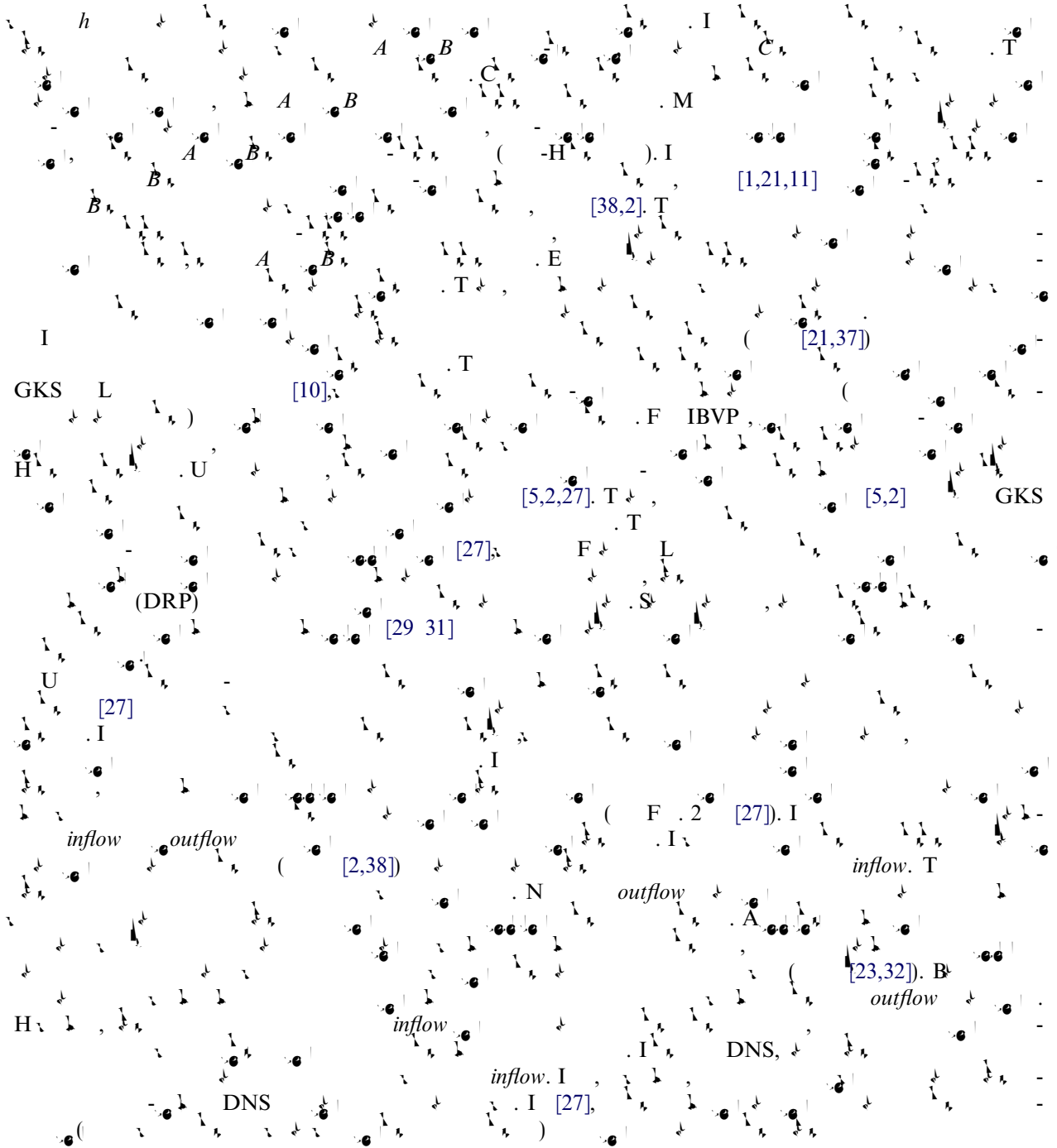
The evolution of a vortex structure in a 2D flow field is studied in this paper. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS. The vortex is initially formed as a vortex tube and then stretches and tilts. The evolution of the vortex is studied using DNS.

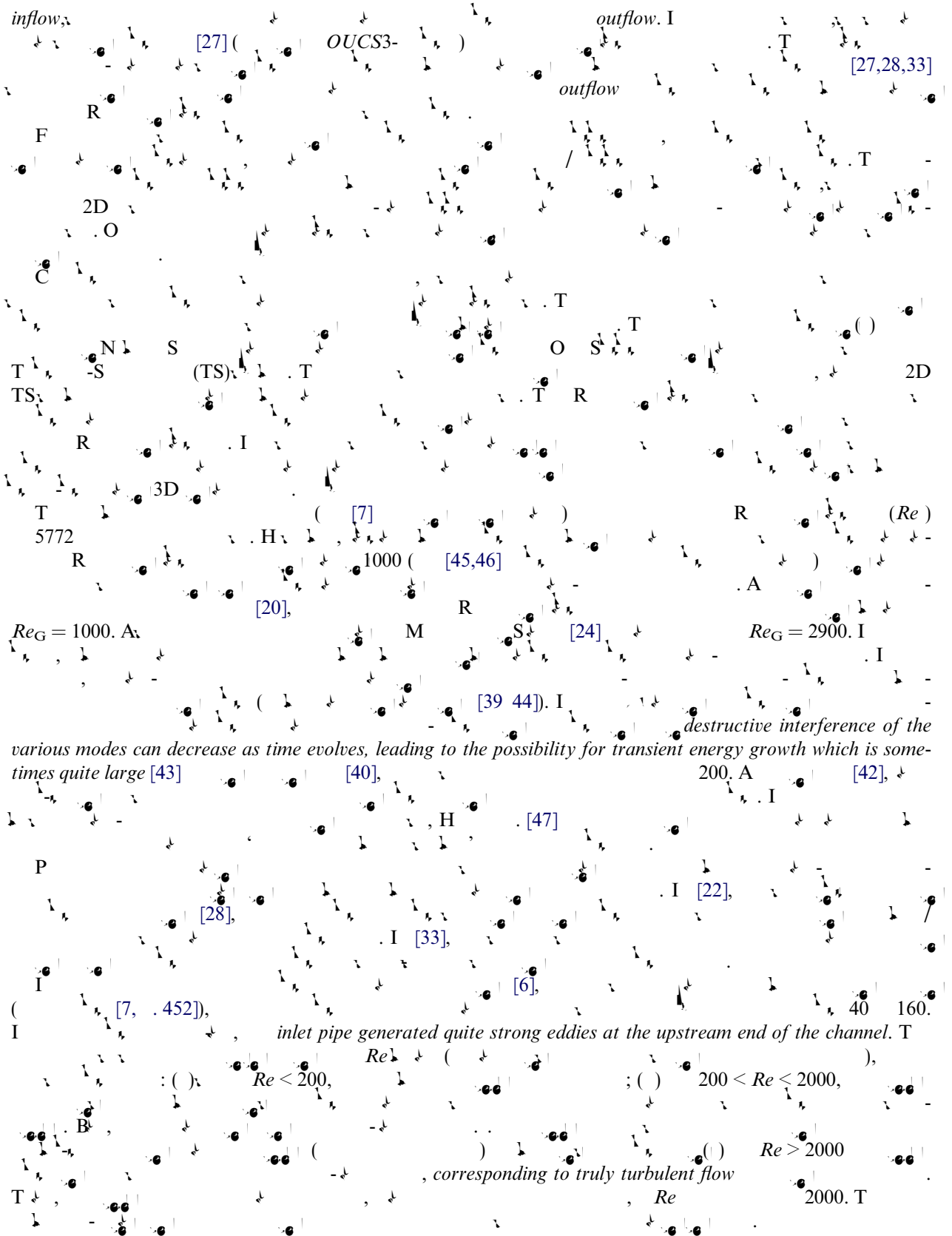
* Corresponding author. Fax: +91 512 590007/5.
E-mail address: asankaran@iitk.ac.in (T.K. Sankaranarayanan).

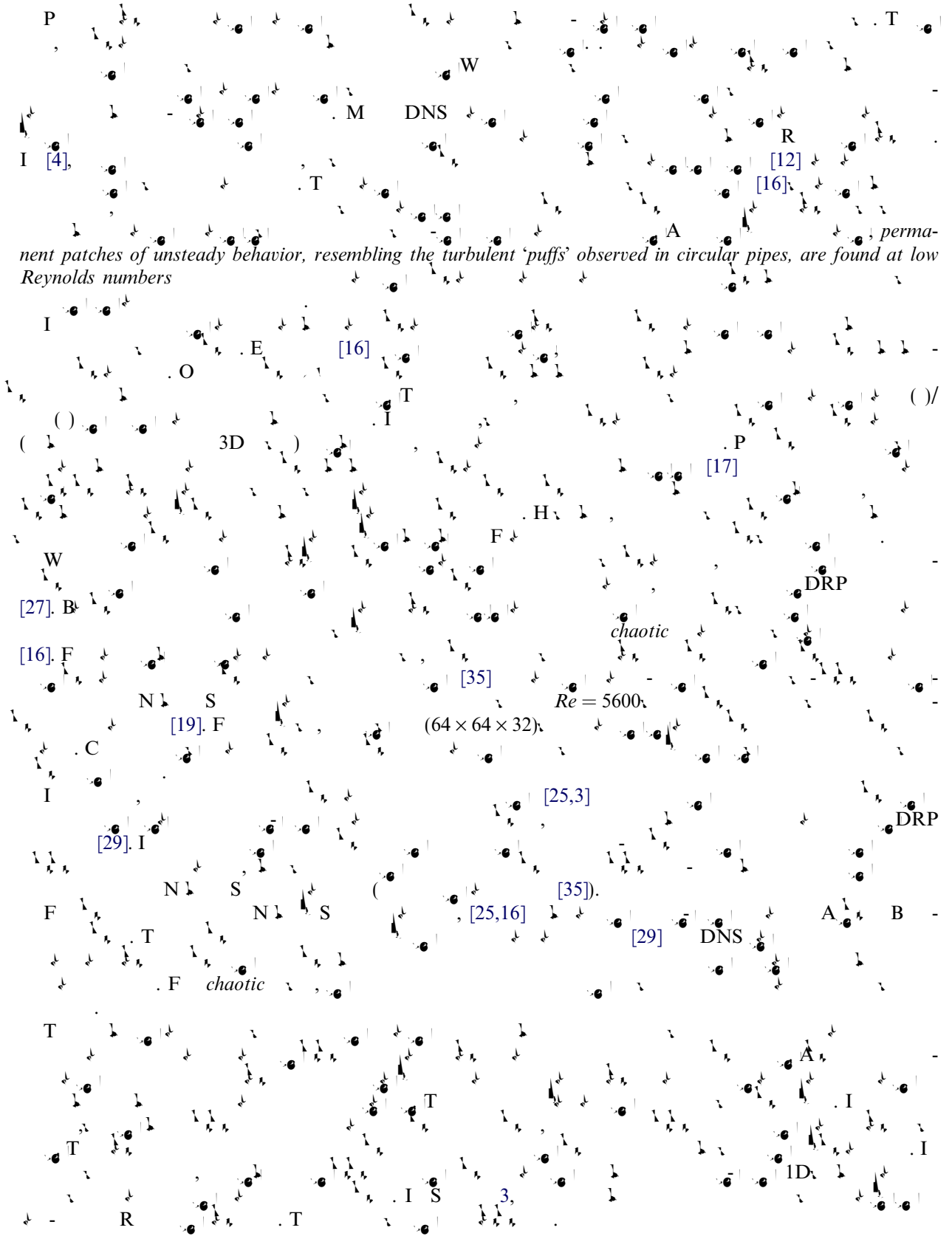
$$F \quad u' \quad u(x_j), j = 1 \dots N, \quad (1)$$

$$[A]\{u'\} = [B]\{u\}.$$

$$\{u'\} = \frac{1}{h}[C]\{u\}, \quad (2)$$







permanent patches of unsteady behavior, resembling the turbulent 'puffs' observed in circular pipes, are found at low Reynolds numbers

I
 O
 E
 [16]
 T
 I
 P
 [17]
 H
 F
 W
 B
 [27]
 F
 [16]
 N
 S
 [19]
 F
 C
 I
 [29]
 I
 N
 S
 N
 S
 F
 T
 N
 S
 F
 chaotic
 T
 T
 T
 I
 S
 3
 R
 T
 A
 B
 I
 I
 ID
 I

[4], [12], [16], [17], [25,3], [25,16], [29], [35]

Re = 5600
 (64 × 64 × 32)

chaotic

DRP

DNS

3D

() /

2. High accuracy symmetrized compact scheme

$$b_{j-1}u'_{j-1} + b_j u'_j + b_{j+1}u'_{j+1} = \frac{1}{h} \sum_{k=-2}^2 a_{j+k}u_{j+k}. \tag{3}$$

$b_j = 1; a_{j\pm 1} = \pm 0.7877868 + \frac{\eta}{30}; a_{j\pm 2} = \pm 0.0458012 + \frac{\eta}{300}; a_j = \frac{-11\eta}{150}; \eta = -2; b_{j\pm 1} = 0.3793894912 \pm \frac{\eta}{60};$
 OUCS3, [27],
 $\gamma_2 = -0.025; \gamma_{N-1} = 0.09; N$
 $j = 1, j = 2, [27];$
 OUCS3

$$u'_1 = \frac{1}{2h} (-3u_1 + 4u_2 - u_3), \tag{4}$$

$$u'_2 = \frac{1}{h} \left[\left(\frac{2\gamma_2}{3} - \frac{1}{3} \right) u_1 - \left(\frac{8\gamma_2}{3} + \frac{1}{2} \right) u_2 + (4\gamma_2 + 1)u_3 - \left(\frac{8\gamma_2}{3} + \frac{1}{6} \right) u_4 + \frac{2\gamma_2}{3} u_5 \right], \tag{5}$$

$\gamma_2 = -0.025; \gamma_{N-1} = 0.09; N$
 $j = N, j = (N-1), j = 3, N-2$
 A, B, C, A, B
 [26,27]

$$u(x_j) = \int_{k_x}^{k_x} U(k) e^{ikx_j} dk \tag{6}$$

$k_x = -k_x = \pi, k_x = 0, 2\pi$
 $u'(x_j) = \int k U(k) e^{ikx_j} dk$

$$u'(x_j) = \int k U(k) e^{ikx_j} dk$$

$$k(x_j) = \sum_{l=1}^N C_{lj} e^{k(x_l - x_j)}. \tag{7}$$

$j, k, kh, 0, \pi, k/k, kh, 0, 2\pi$
 [21,27]

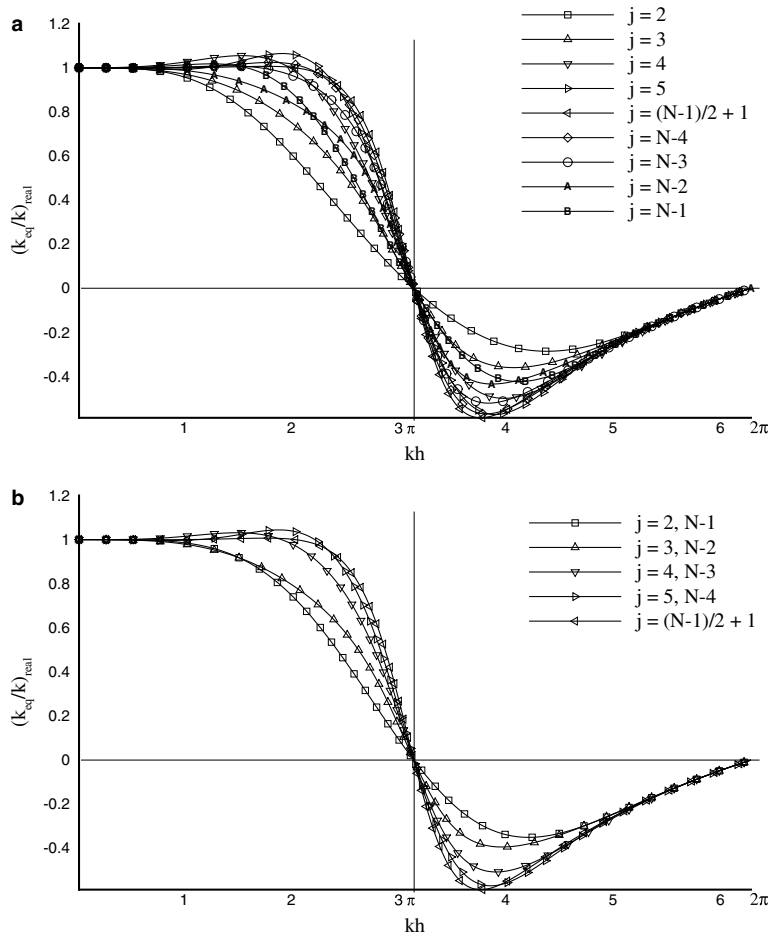
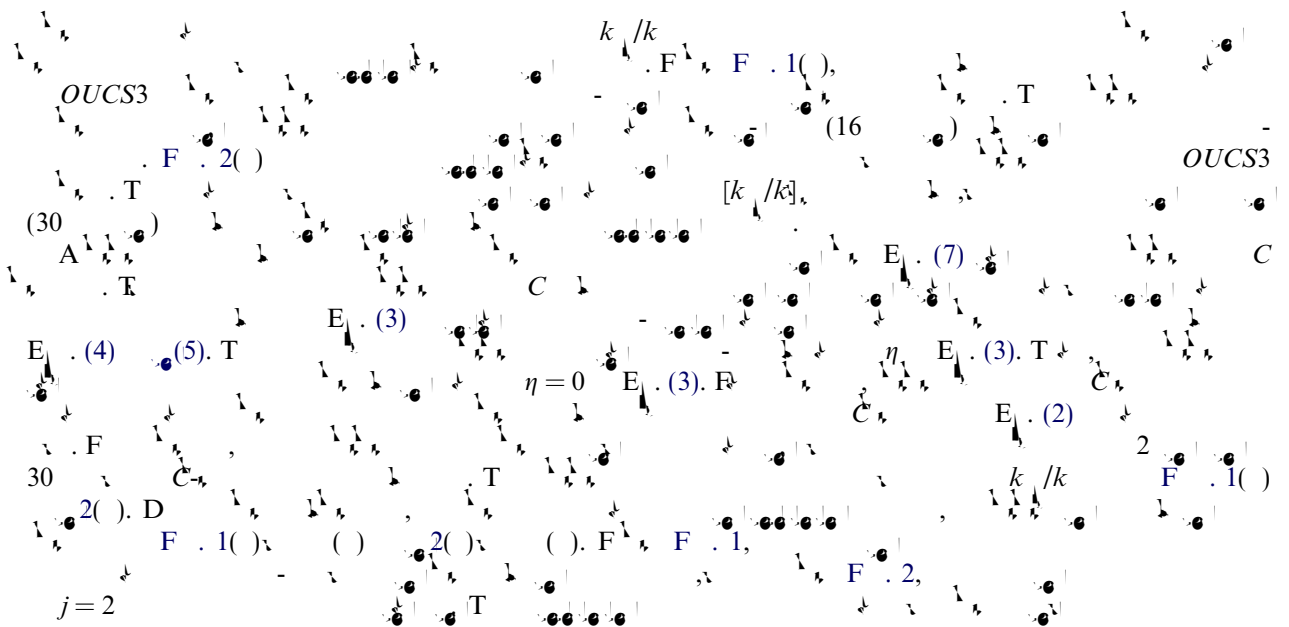


Fig. 1. Real part of the ratio (k_{eq}/k) versus kh for OUCS3 (□) and S-OUCS3 (●) with $N = 31$. The legend in (a) and (b) indicates the values of j for the different curves.



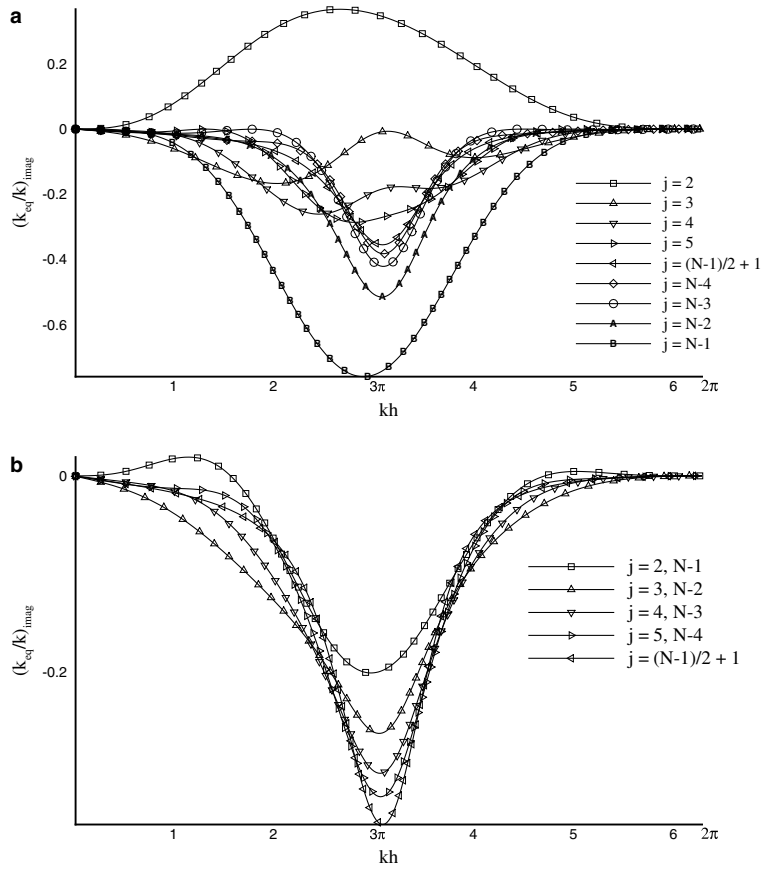
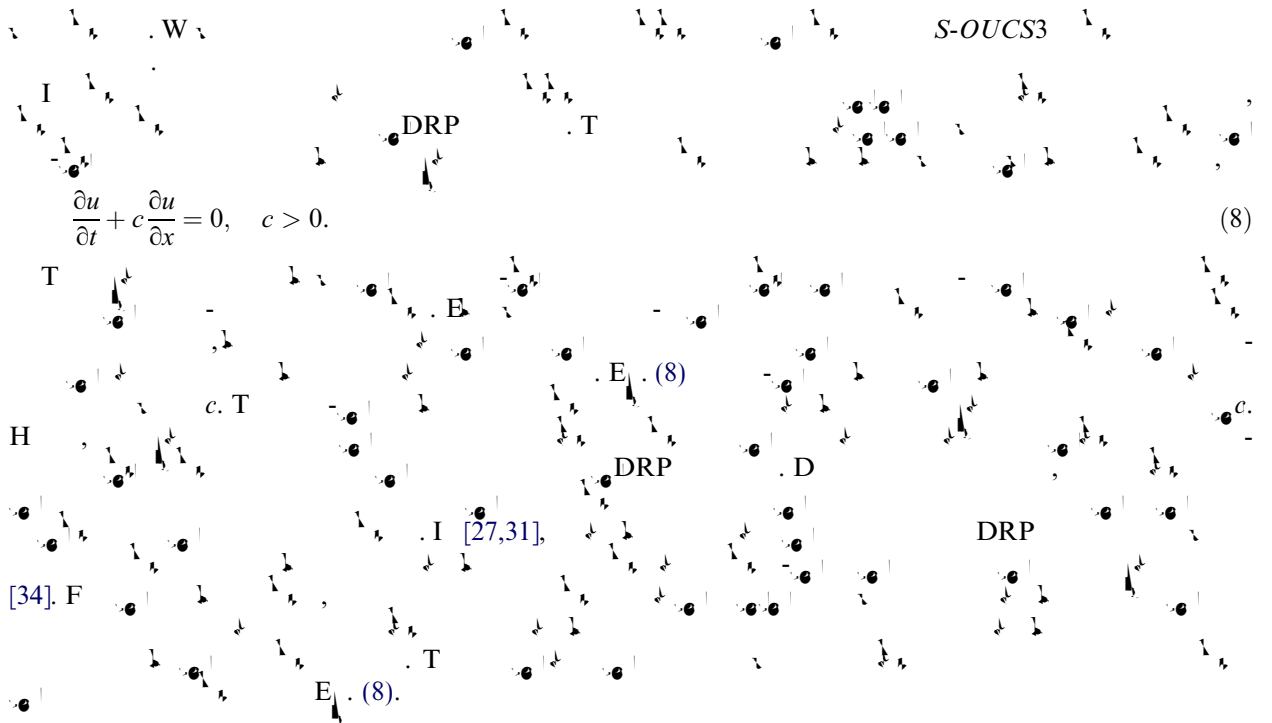


Fig. 2. (a) $(k_{co}/k)_{img}$ versus kh for OUCS3. (b) $(k_{co}/k)_{img}$ versus kh for S-OUCS3. $N = 31$.



C

$$u_m^0 = u(x_m, t = 0) = \int A_0(k) e^{ikx_m} dk \quad (9)$$

T

$$u(x, t) = \int A_0(k) e^{k(x-ct)} dk \quad (10)$$

T

N

O

[29]

$$u_m^n = \int \hat{U}(k, t^n) e^{ikx_m} dk$$

$$G(k) = \frac{\hat{U}(k, t^{n+1})}{\hat{U}(k, t^n)}$$

$$u_m^n = u(x_m, t^n) = \int A_0(k) (G_r^2 + G_i^2)^{n/2} e^{k(x_m - n\beta)} dk \quad (11)$$

E

(10)

(11),

F

E

(11),

c_N [29,31]

$$G(k) = G_r(k) + G_i(k) \quad \beta = -G_i/G_r$$

$$k(x_m - n\beta) = k(x_m - c_N t^n)$$

$$\frac{\hat{c}_N(k)}{c} = \frac{\beta}{\omega \Delta t} \quad (12)$$

$\omega = ck$ T

[29,31]

$$\omega_N = \hat{c}_N k$$

$$\frac{V_{gN}(k)}{c} = \frac{1}{N_c h} \frac{\beta}{k} \quad (13)$$

F

DNS,

F

$V_{gN}(k)$

E

(8),

(CD₂)

[29]

$$G_r = \frac{(|G| = 1)}{2(khN_c)}$$

$$(\hat{c}_N(k) = c)$$

$$k/k, G(k)$$

$$G(kh, N_c) = [1 + N_c^2 (kh)^2]^{1/2} - \beta$$

$$\beta = -N_c (kh)$$

F

E

(7),

$$\frac{\partial u_j}{\partial x} = \frac{1}{h} \sum_{l=1}^N C_{lj} e^{k(x_l - x_j)} u_j \quad (14)$$

U

E

(8),

$$\frac{\partial u_j}{\partial t} + \frac{cu_j}{h} \sum C_{lj} [k(x_l - x_j) + k(x_l - x_j)] = 0 \quad (15)$$

T

F

E

$G_j(k)$,

$$G_j(kh, N_c) = G_{jr} + G_{ji} = 1 - N_c \sum_{l=1}^N C_{lj} [k(x_l - x_j) + k(x_l - x_{j+1})] \quad (16)$$

$$(\beta_j) = -G_{ji}/G_{jr} \quad (17)$$

[31],

DRP

$$L(u) = -c \frac{\partial u}{\partial x} \quad (18)$$

$$S-1: u^{(1)} = u^{(n)} + \frac{\Delta t}{2} L[u^{(n)}],$$

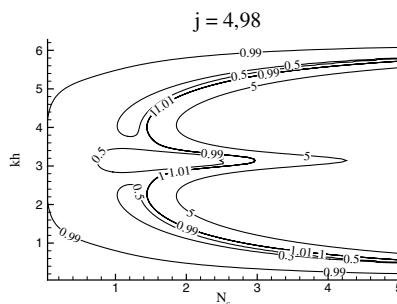
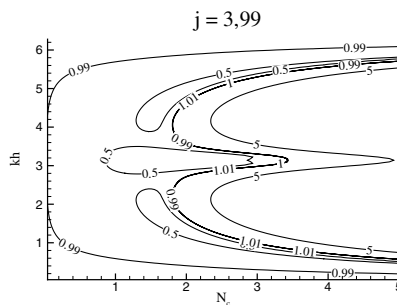
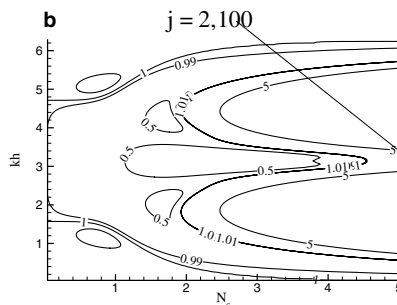
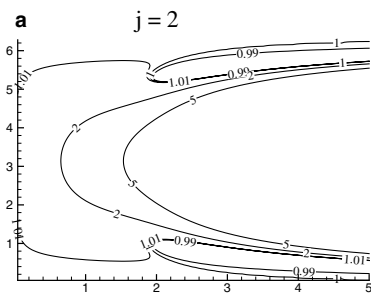
$$S-2: u^{(2)} = u^{(n)} + \frac{\Delta t}{2} L[u^{(1)}],$$

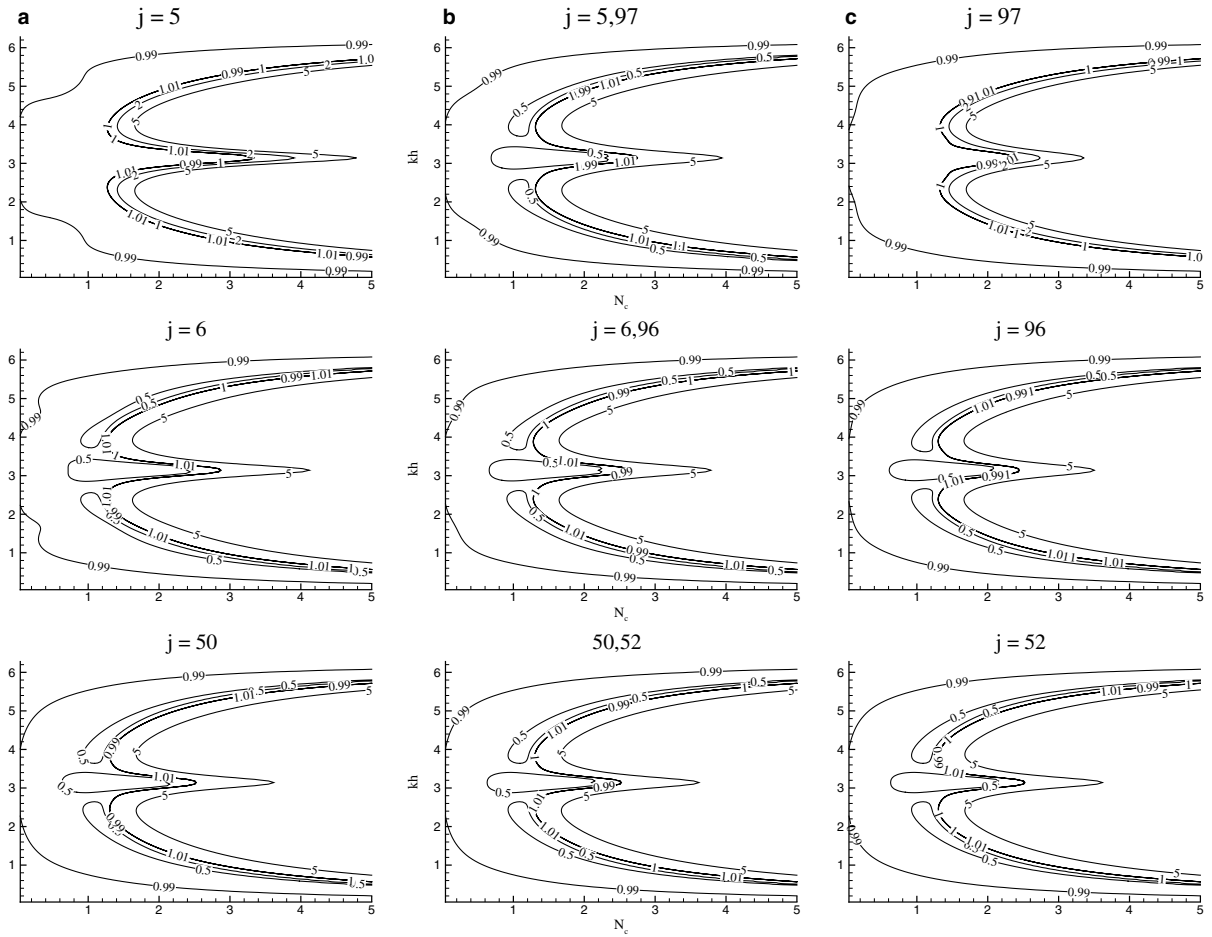
$$S-3: u^{(3)} = u^{(n)} + \Delta t L[u^{(2)}],$$

$$S-4: u^{(n+1)} = u^{(n)} + \frac{\Delta t}{6} \{L[u^{(n)}] + 2L[u^{(1)}] + 2L[u^{(2)}] + L[u^{(3)}]\}.$$

T RK4

[26]

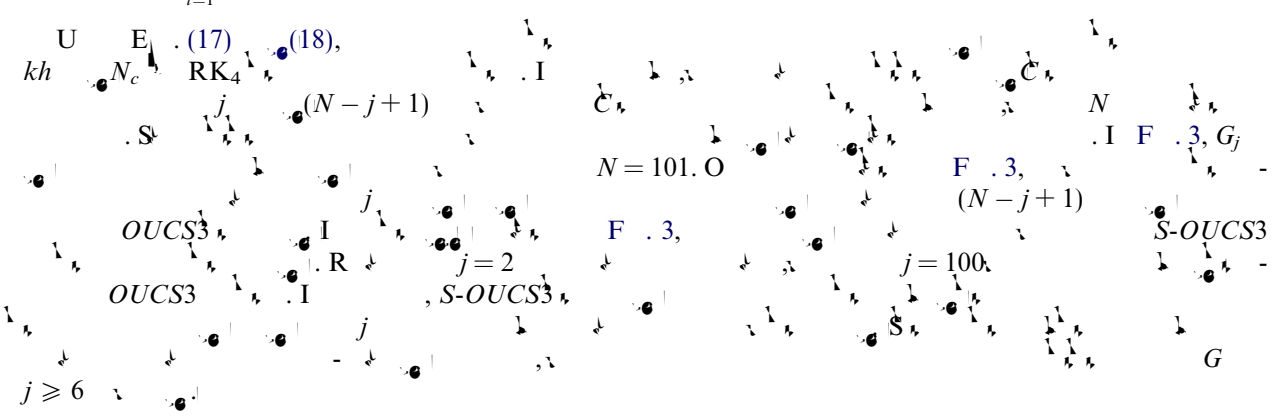




F . 3 (continued)

$$G_j(kh, N_c) = 1 - A_j + \frac{A_j^2}{2} - \frac{A_j^3}{6} + \frac{A_j^4}{24}, \tag{17}$$

$$A_j = N_c \sum_{l=1}^N C_{lj} k^{(x_l - x_j)}. \tag{18}$$



4,

kh

3×10^{-5}

0.008. I

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

J

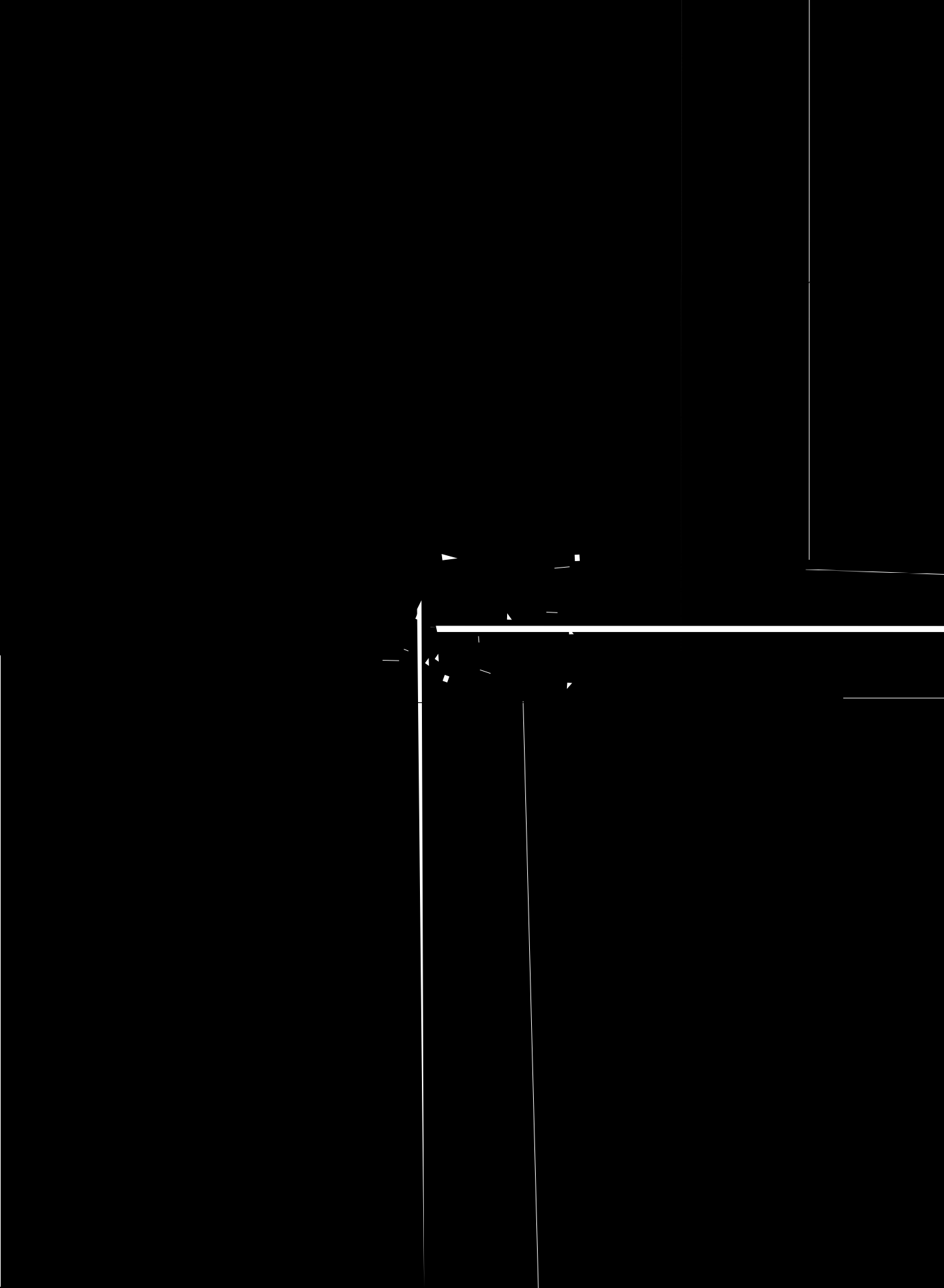
J

J

(V

UCS3 () S-OUCS3

•••••



I \quad E_1 \cdot (21) \quad \lambda_1 \quad \lambda_2

$$\lambda_1 = F^{-\eta}, \tag{22}$$

$$\lambda_2 = H^{-\Gamma}, \tag{23}$$

$$u_m^n = \int M(k)[F]^{n(kx_m+mn)} \cdot k + \int N(k)[H]^{n(kx_m+n\Gamma)} \cdot k, \tag{24}$$

$$M(k) \quad N(k)$$

[31]

E_1 \cdot (19)

[21] W
[21]

S-OUCS3

$$\frac{\partial^2 u}{\partial x^2} \quad E_1 \cdot (19)$$

[21]

E_1 \cdot (19). T

$$j = 1: \quad u_1'' + 11u_2'' = (13u_1 - 27u_2 + 15u_3 - u_4)/h^2, \tag{25}$$

$$j = 2: \quad u_1'' + 10u_2'' + u_3'' = 12(u_3 - 2u_2 + u_1)/h^2, \tag{26}$$

$$3 \leq j \leq N - 2: \quad \alpha u_{j-1}'' + u_j'' + \alpha u_{j+1}'' = \frac{b}{4h^2}(u_{j-2} + 2u_j + u_{j+2}) + \frac{a}{h^2}(u_{j-1} - 2u_j + u_{j+1}). \tag{27}$$

O \quad j = N \quad N - 1 \quad E_1 \cdot (25) \quad (26), \quad I [21],

$$\alpha = 2/11; a = 12/11 \quad b = 3/11 \quad E_1 \cdot (27). T$$

$$-\int k^2 U(k) \quad kx \quad k^2 \quad \frac{\partial^2 u}{\partial x^2} \quad E_1 \cdot (6), \quad u'' =$$

$$\frac{k^2}{k^2} \quad M \quad [21] \quad S-OUCS3 \quad F \cdot 5()$$

$$0 \quad 2\pi \quad kh = \pi, \quad S-OUCS3 \quad S-OUCS3$$

$$0.7 \quad S-OUCS3 \quad S-OUCS3 \quad P_j^2 \quad E_1 \cdot (21)$$

$$A \quad E_1 \cdot (25) \quad (27) \quad S-OUCS3 \quad I \quad F \cdot 5() \quad (), \quad T$$

$$F \quad ABC \quad ABC \quad (kh \quad N_c) \quad T$$

$$ABC \quad N \quad ABC, \quad I$$

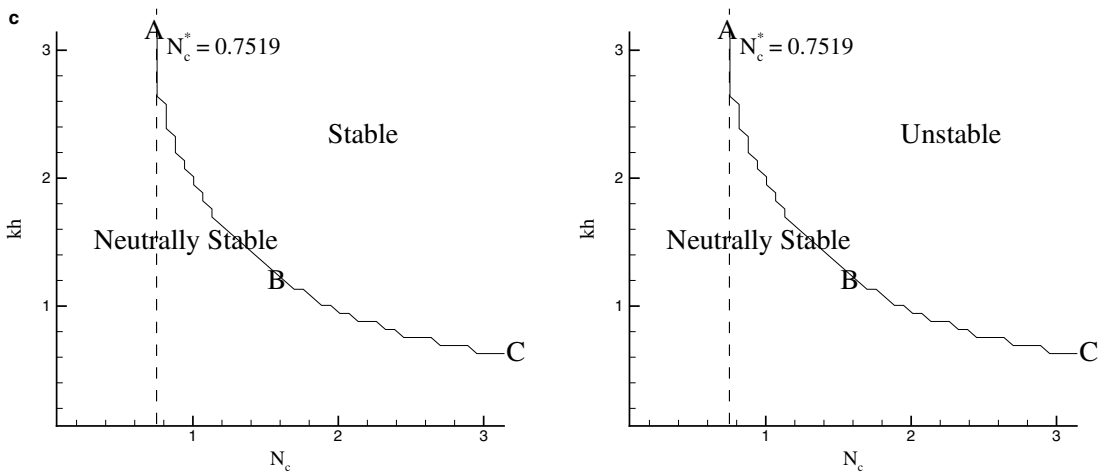
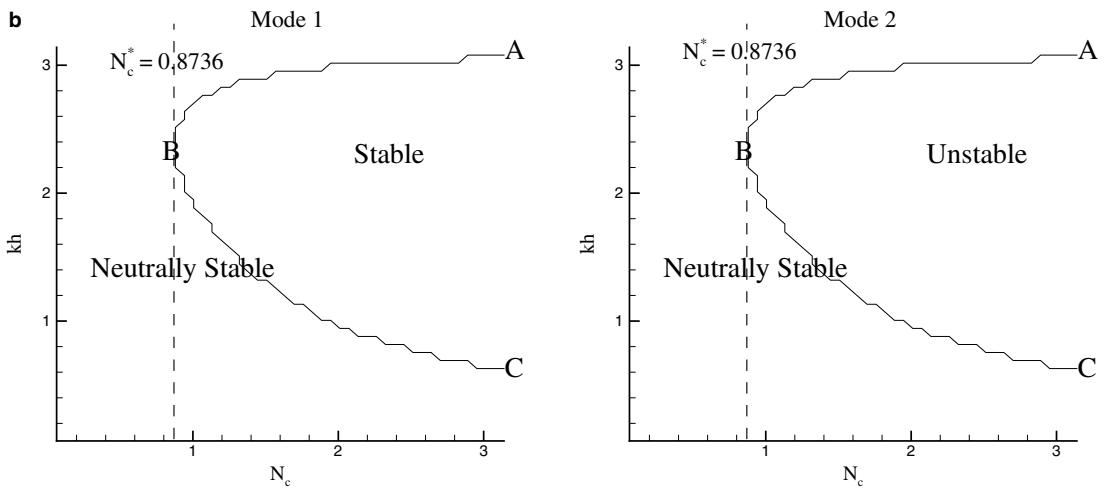
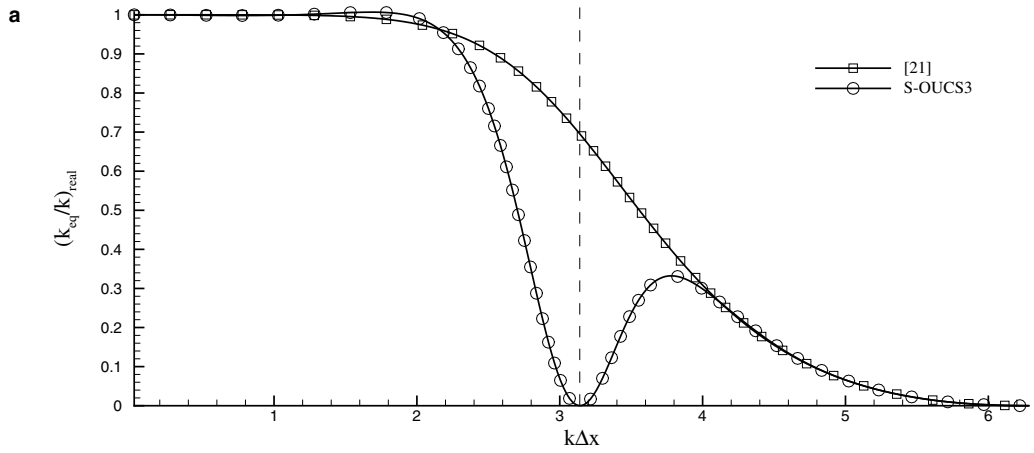
$$F \quad S-OUCS3 \quad CFL \quad (N_c^*) \quad I \quad kh$$

$$N_c^* = 0.7519. \quad T \quad 0.8736 \quad T \quad S-$$

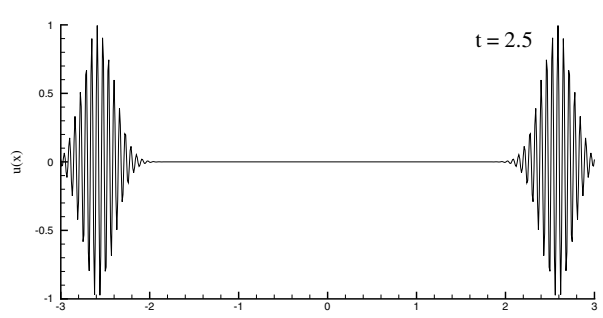
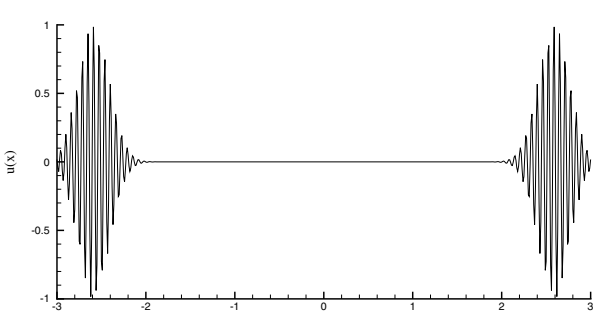
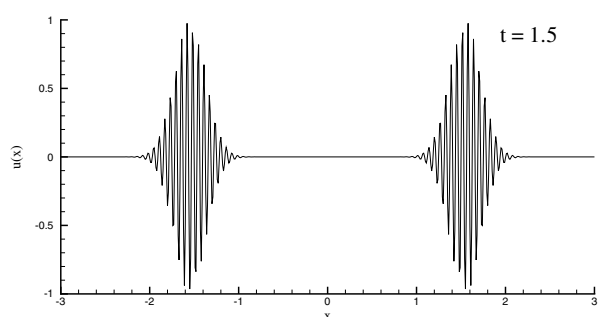
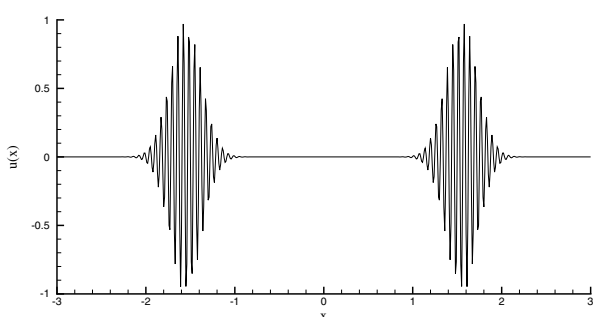
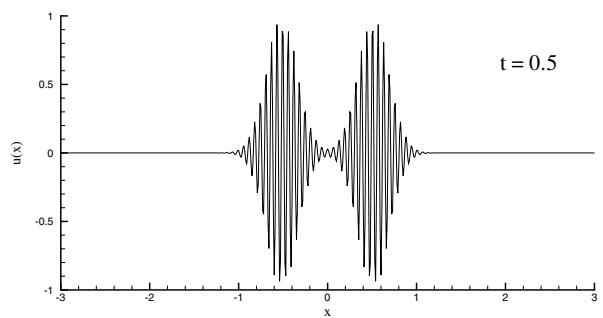
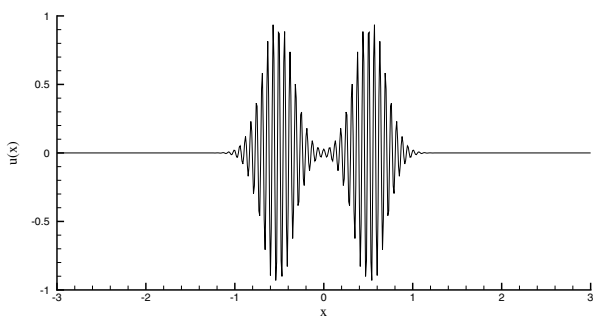
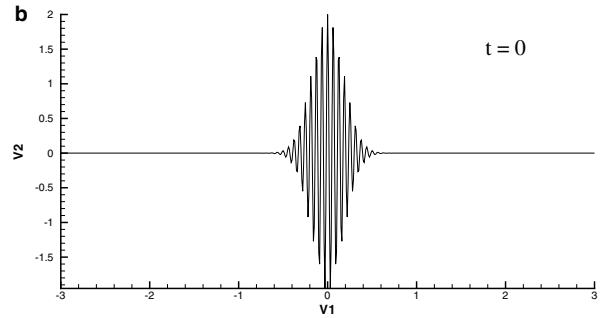
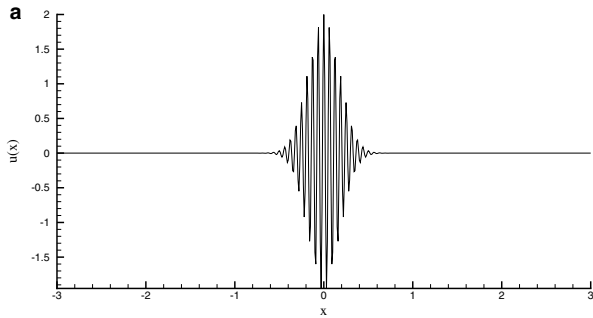
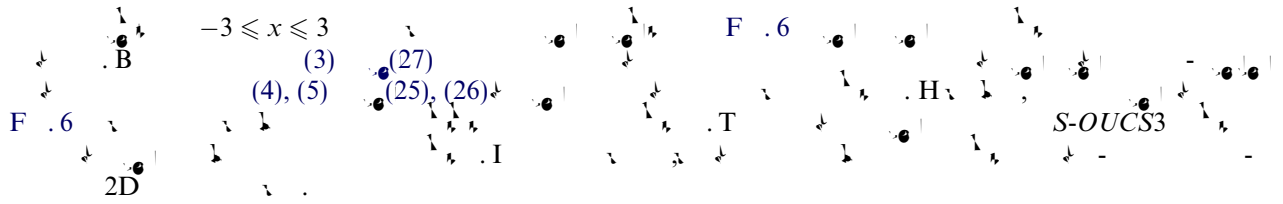
$$OUCS3 \quad kh \quad 1.8 (\quad F \cdot 4 \quad) \quad T$$

$$W \quad [21] \quad E_1 \cdot (19) \quad T$$

$$u_0(x) = 2^{-16x^2} k_0 x. \tag{28}$$



F . 5. N_c^* S-OUCS3 [21], $(k_{eq}/k)_{real}$ A, $(|G|)$
 T $k_0h = 1. F$ F . 4 . 5. T $k_0h,$
 $c = 1. T$



F .6. C, (19) S-OUCS3 [21]

4. Transitional flow in a channel

W N S (ω) (VTE):

$$\nabla^2 \psi = -\omega \tag{29}$$

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{V} \cdot \nabla) \vec{\omega} = \frac{1}{Re} \nabla^2 \vec{\omega} \tag{30}$$

T A T

$$\frac{\partial}{\partial \xi} \left[\frac{h_2}{h_1} \frac{\partial \psi}{\partial \xi} \right] + \frac{\partial}{\partial \eta} \left[\frac{h_1}{h_2} \frac{\partial \psi}{\partial \eta} \right] = -h_1 h_2 \omega \tag{31}$$

$$h_1 h_2 \frac{\partial \omega}{\partial t} + h_2 u \frac{\partial \omega}{\partial \xi} + h_1 v \frac{\partial \omega}{\partial \eta} = \frac{1}{Re} \left[\frac{\partial}{\partial \xi} \left(\frac{h_2}{h_1} \frac{\partial \omega}{\partial \xi} \right) + \frac{\partial}{\partial \eta} \left(\frac{h_1}{h_2} \frac{\partial \omega}{\partial \eta} \right) \right], \tag{32}$$

$$h_1^2 = \left(\frac{\partial x}{\partial \xi} \right)^2 + \left(\frac{\partial y}{\partial \xi} \right)^2$$

$$h_2^2 = \left(\frac{\partial x}{\partial \eta} \right)^2 + \left(\frac{\partial y}{\partial \eta} \right)^2$$

I F

$$x(\xi) = L \left[1 - \frac{[\beta_1(1-\xi)]}{[\beta_1]} \right] \tag{33}$$

T [4]

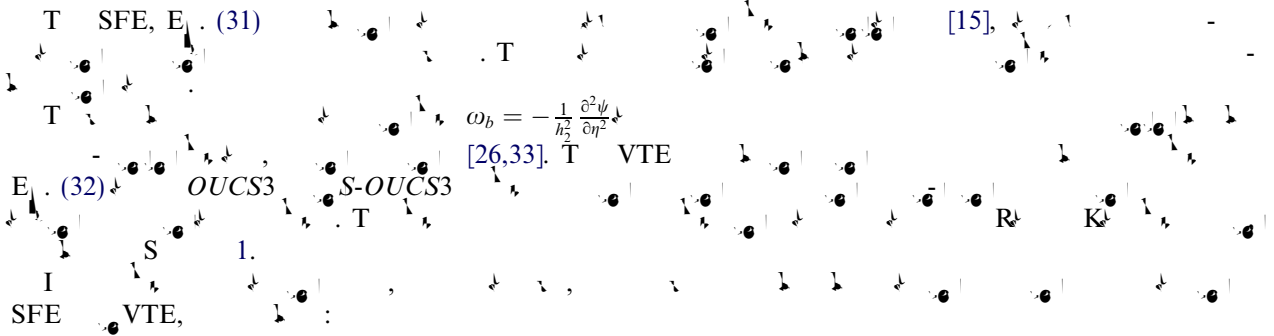
$$0 \leq \xi \leq 1, \beta_1 = 1.55, h_1 = x_\xi, f = 1/h_1$$

$$f = \frac{[\beta_1]}{4\beta_1 L} [2\beta_1(1-\xi) + -2\beta_1(1-\xi) + 2].$$

T L f

$$\hat{F}(k) = \frac{(\beta_1)}{2\beta_1 L} \left[\delta(k) + \frac{1}{2} \delta(k - 2\beta_1) [2\beta_1 - 2\beta_1] + \frac{1}{2} \delta(k + 2i\beta_1) [2\beta_1 + 2\beta_1] \right].$$

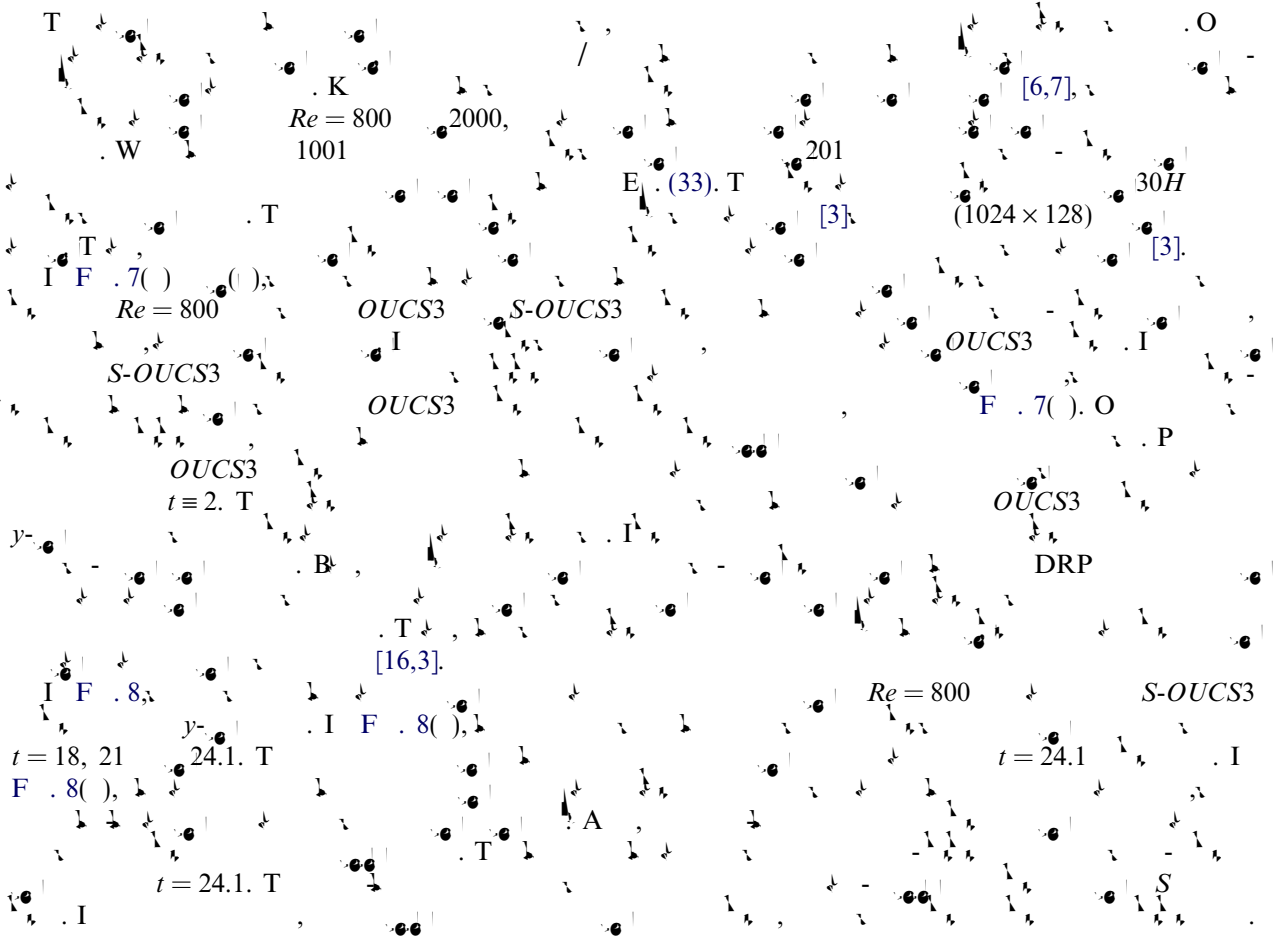
T, $\hat{F}(k)$, $k=0$, $2\beta_1$, I, W, S-OUCS3, OUCS3, η , ξ , VTE.



$$\frac{\partial u_x}{\partial t} + u_c \frac{\partial u_x}{\partial x} = 0 \tag{34}$$

$$\frac{\partial \omega}{\partial t} + u_c \frac{\partial \omega}{\partial x} = 0, \tag{35}$$

4.1. Establishment of equilibrium flow



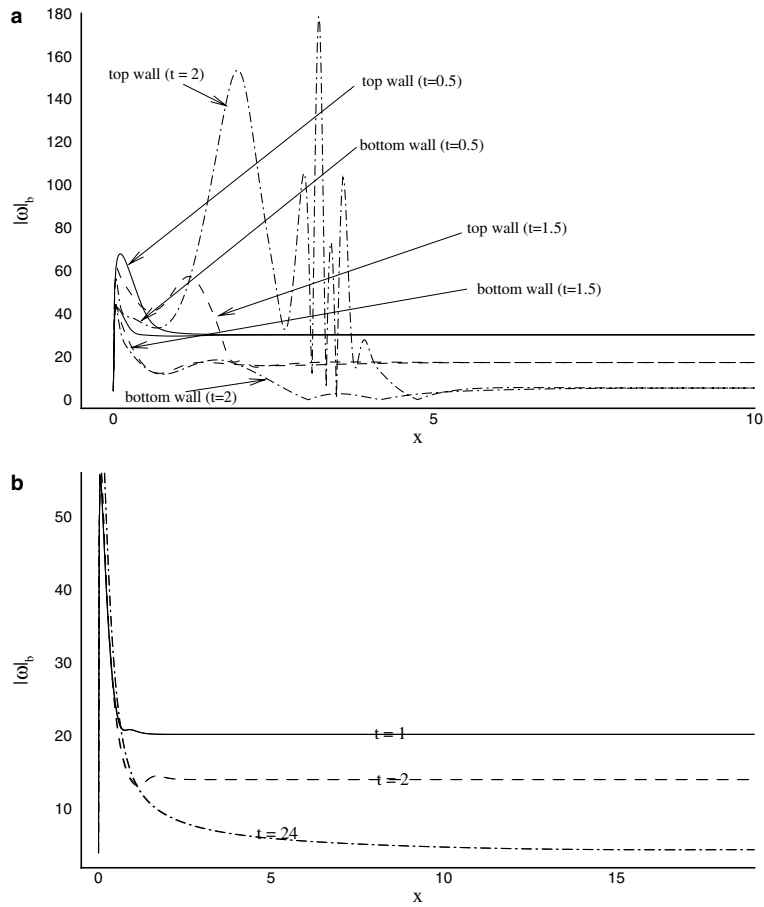
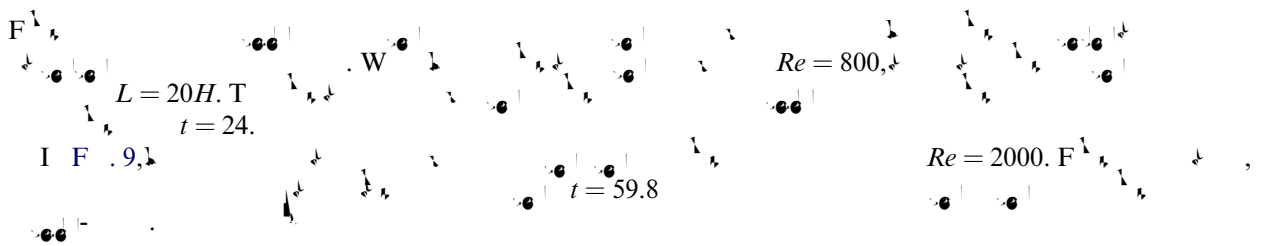
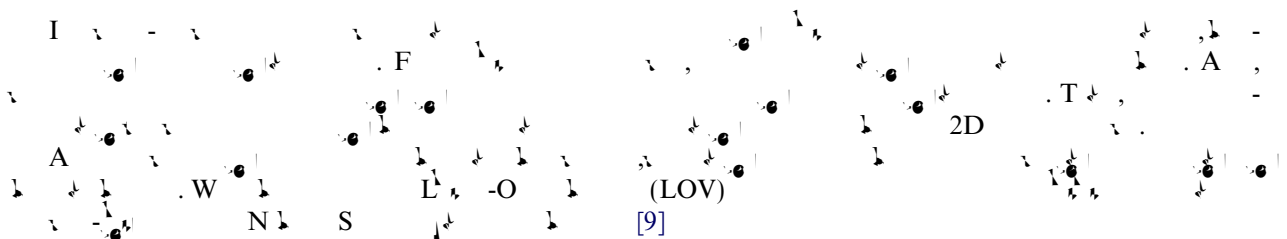


Fig. 7. Vorticity profiles at the top wall (—) and bottom wall (---) for $Re = 800$ and $Re = 2000$ at various times. (—) OUCS3, (---) S-OUCS3.

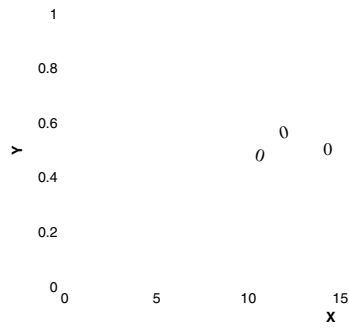
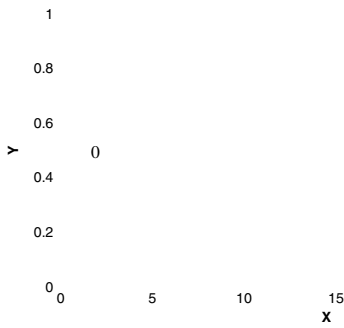


4.2. Receptivity of channel flow to single viscous vortex

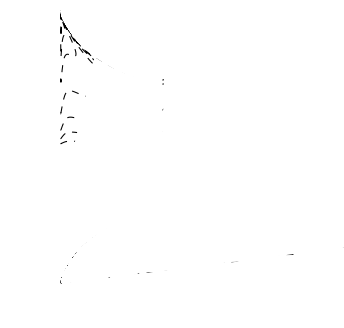


$$\omega^*(r^*, t^*) = \frac{\Gamma_0^*}{4\pi\nu t^*} \left(\frac{-(r^*)^2}{4\nu t^*} \right)$$

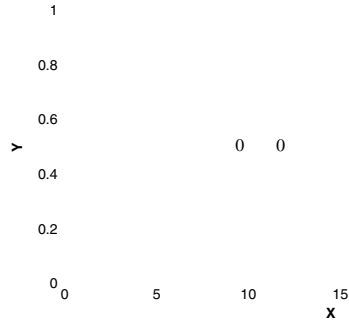
t = 4



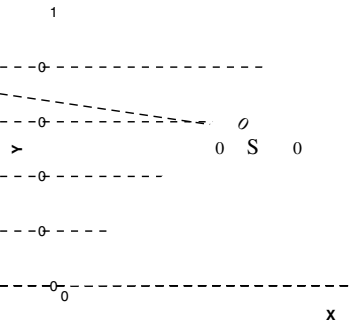
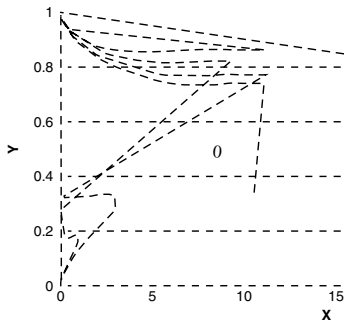
t = 20

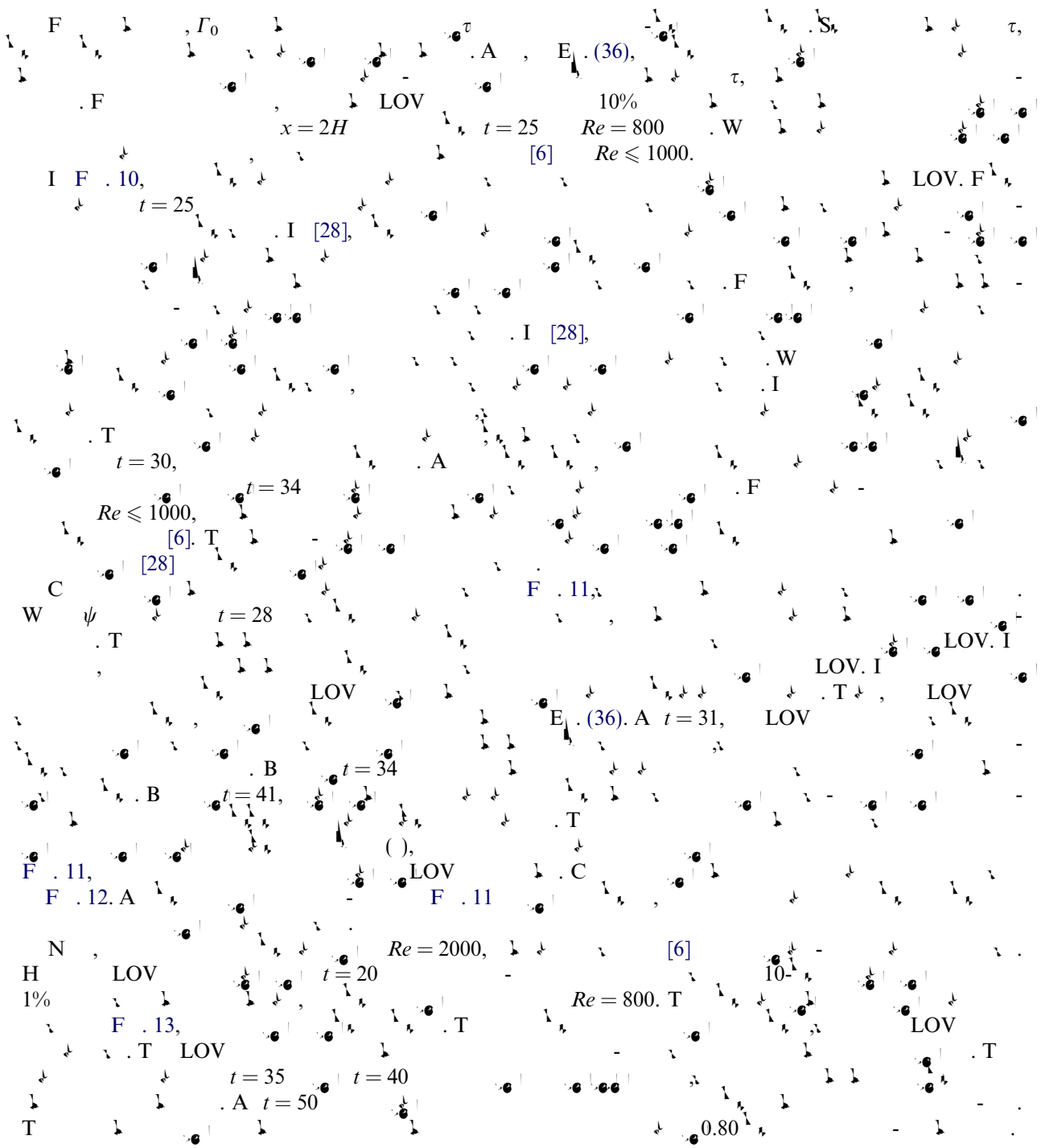


t = 12

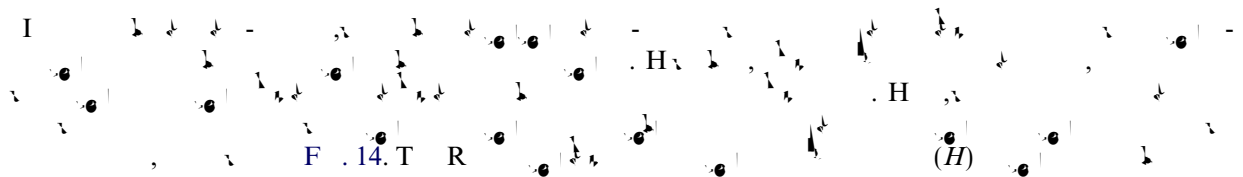


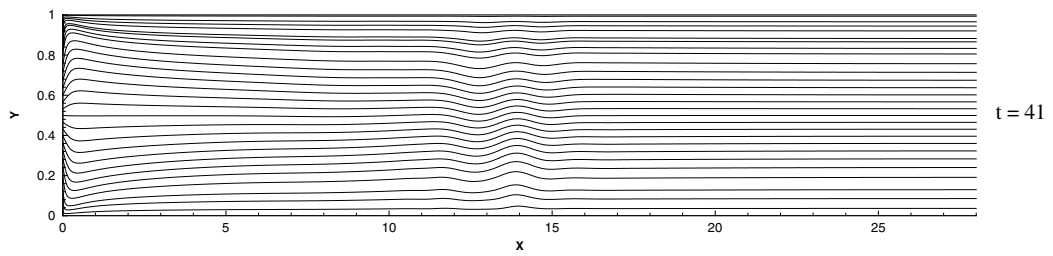
t = 24.1

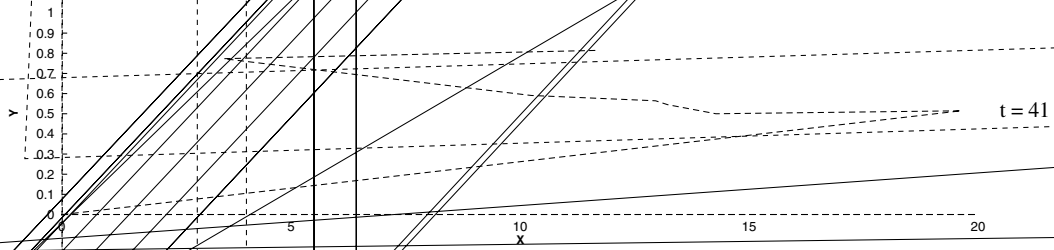
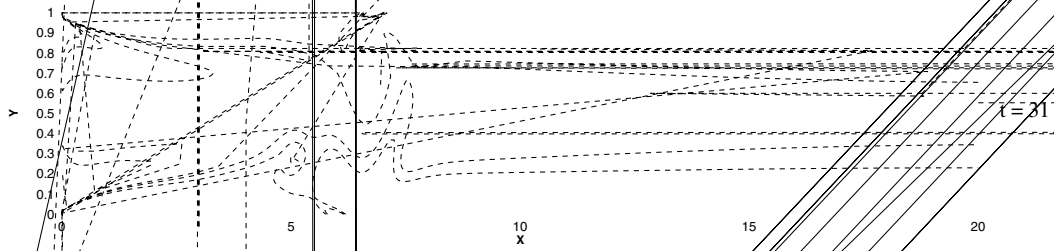




4.3. Transition of a channel flow created by vortex street







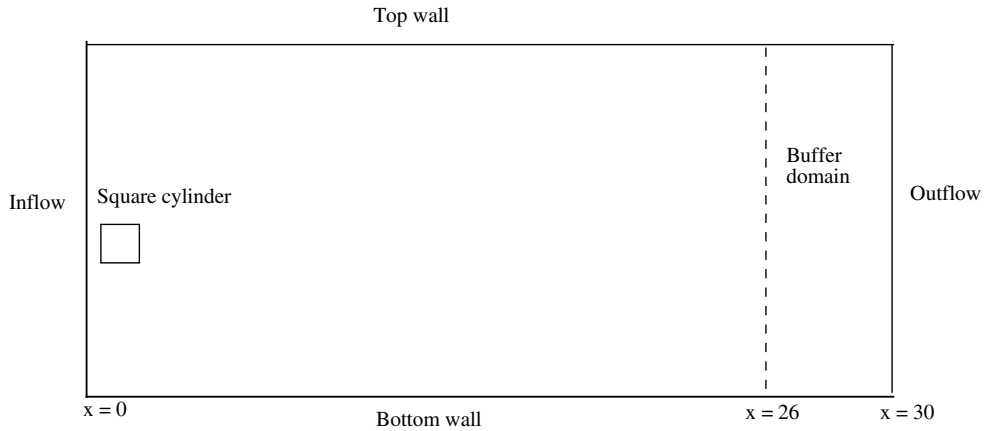
1
0.8
0.6
0.4
0.2
0
0
y

1
0.8
0.6
0.4
0.2
0
0
y

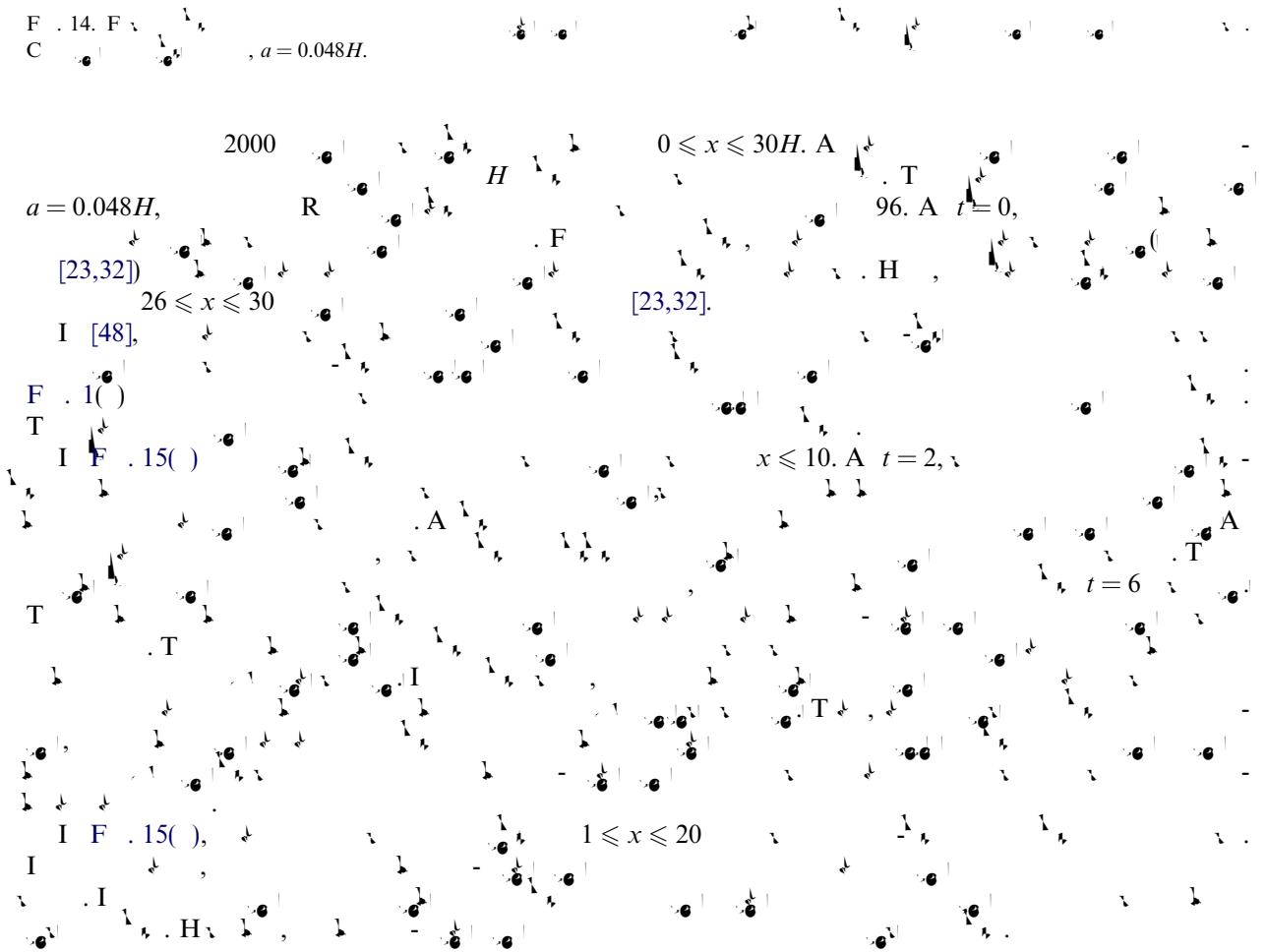
1
0.8
0.6
0.4
0.2
0
0
y

F . 13. V

• • •

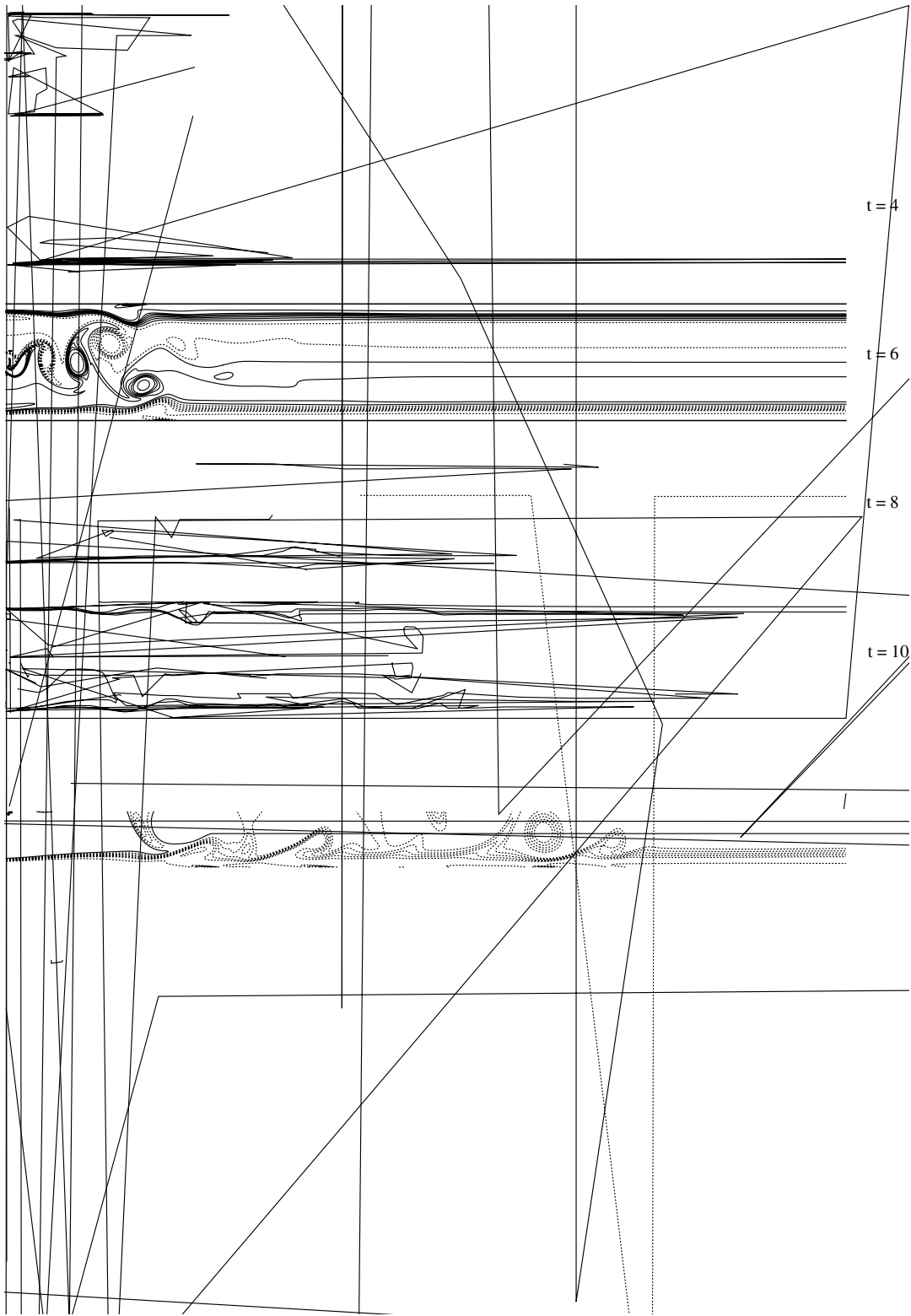


$F = 14$, $C = 0.14$, $a = 0.048H$.



5. Summary





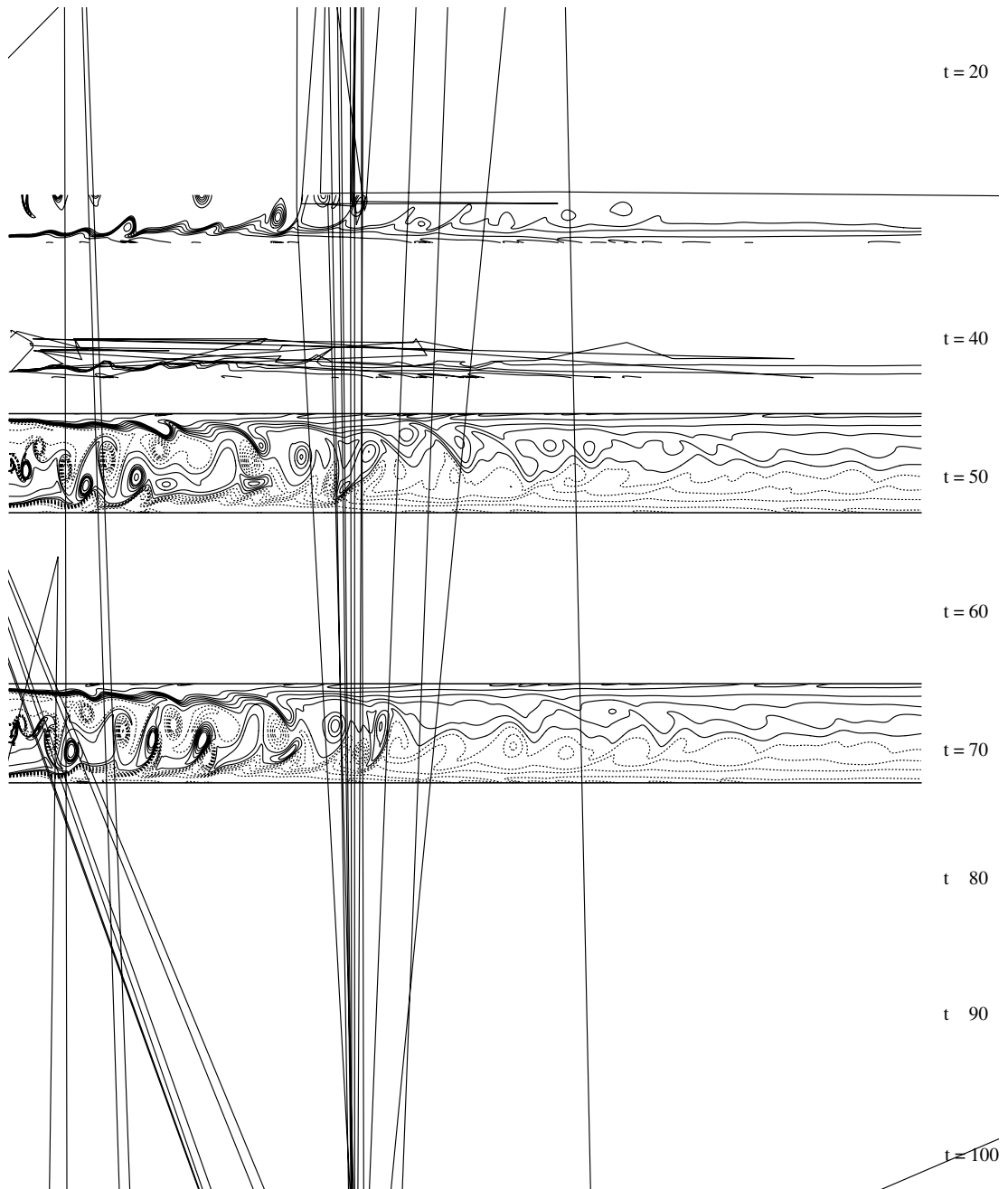


Fig. 15. (a) DNS, (b) TR, (c) 2D, (d) I, (e) B. $Re = 2000$, $\Delta x = 1$, $\Delta y = 1$, $\Delta z = 1$, $\Delta t = 0.1$, $N_x = 128$, $N_y = 128$, $N_z = 64$, $N_t = 100$.

References

- [1] Y. A. Ibragimov, J. Comput. Phys. 24 (1977) 10.
- [2] N.A. Abdalrhman, K. Sengupta, J. Comput. Phys. 127 (1996) 27.
- [3] T.R. Bhowmik, P. Maiti, R. T. Sengupta, J. Fluid Mech. 447 (2001) 179.
- [4] T.R. Bhowmik, O.M. Aouf, J. Fluid Mech. 499 (2004) 183.
- [5] M.H. Choudhury, D.G. Goussis, S.A. Orszag, J. Comput. Phys. 108 (1993) 272.
- [6] S.J. Drell, C.M. Wolfe, P. Resnikoff, SIAM J. Numer. Anal. 119 (1982) 92.
- [7] P.G. Drazin, W.H. Reid, Hydrodynamic Stability, Cambridge University Press, Cambridge, UK, 1981.
- [8] D.G. Goussis, J.S. Sengupta, J. Comput. Phys. 138 (1997) 617.
- [9] S.I. Ghosh, S.I. Ghosh (Ed.), Fluid Velocity, Kinematics and Dynamics, Taylor & Francis, London, 1995.
- [10] B.G. Ghosh, H.O. Kulkarni, A.S. Mahajan, J. Comput. Phys. 26 (1972) 649.
- [11] Z.H. Gao, S.T. Gao, J. Comput. Phys. 114 (1994) 265.
- [12] D.S. Hesthaven, P. Sengupta, J. Comput. Phys. 30 (1987) 2914.
- [13] R.S. Hesthaven, J. Comput. Phys. 19 (1975) 90.
- [14] R.H. Hesthaven, E. Tadmor, J. Comput. Phys. 158 (2000) 51.
- [15] A.J. Hesthaven, G.M. Maiti, I. J. Numer. Methods 12 (1978) 141.
- [16] J. Hesthaven, J. Fluid Mech. 218 (1990) 265.
- [17] J. Hesthaven, P. Maiti, R. Maiti, J. Fluid Mech. 177 (1987) 133.
- [18] Z.K. Kang, N.K. Aouf, SIAM J. Numer. Anal. 4 (1966) 1233.
- [19] H.P. Kang, H.E. Aouf, P. Fluid Mech. 22 (1979) 1233.
- [20] L.D. Landau, E.M. Lifshitz, Fluid Mechanics, 6th Edition, Pergamon Press, Oxford, UK, 1959.
- [21] S.K. Lele, J. Comput. Phys. 103 (1992) 16.
- [22] T.T. Lele, T.K. Sengupta, M. Choudhury, E. Fluid Mech. 37 (2004) 47.
- [23] Z. Lele, C. Lele, J. Comput. Phys. 23 (1994) 955.
- [24] D. Maiti, J.T. Sengupta, P. Resnikoff, SIAM J. Numer. Anal. 208 (1982) 517.
- [25] P. Maiti, J. Kang, J. Fluid Mech. 118 (1982) 341.
- [26] T.K. Sengupta, R. Sengupta, J. Fluid Mech. 470 (2002) 1.
- [27] T.K. Sengupta, G. Goussis, S. D. Sengupta, J. Comput. Phys. 192 (2) (2003) 677.
- [28] T.K. Sengupta, S. D. Sengupta, J. Fluid Mech. 493 (2003) 277.
- [29] T.K. Sengupta, A. D. Sengupta, J. S. Comput. Phys. 21 (1) (2004) 225.
- [30] T.K. Sengupta, S.G. Sengupta, A. D. Sengupta, J. S. Comput. Phys. 21 (3) (2004) 253.
- [31] T.K. Sengupta, S.K. Sengupta, A. D. Sengupta, J. S. Comput. Phys. 25 (2) (2005).
- [32] T.K. Sengupta, M. Choudhury, Z.Y. Wang, K.S. Yoon, J. Fluid Mech. 16 (1) (2002) 15.
- [33] T.K. Sengupta, A. D. Sengupta, J. Fluid Mech. 529 (2005) 147.
- [34] L.N. Tadmor, SIAM Review 24 (2) (1982) 113.
- [35] R.W.C.P. Vignati, A.E.P. Vignati, J. Comput. Phys. 187 (2003) 343.
- [36] D.W. Zang, SIAM J. Numer. Anal. 22 (2) (2000) 476.
- [37] D.W. Zang, H. L. Sengupta, H.M. Sengupta, SIAM J. Numer. Anal. 17 (1996) 328.
- [38] X. Zang, J. Comput. Phys. 144 (1998) 622.
- [39] K.M. Bhowmik, B.F. Foy, P. Fluid Mech. 4 (1992) 1637.
- [40] S.C. Ramesh, D.S. Hesthaven, J. Fluid Mech. 252 (1993) 209.
- [41] D.S. Hesthaven, S.C. Ramesh, P. Fluid Mech. 6 (1994) 1396.
- [42] L.N. Tadmor, A.E. Tadmor, S.C. Ramesh, T.A. D. Sengupta, SIAM J. Numer. Anal. 261 (1993) 578.
- [43] M. Hesthaven, T.R. Bhowmik, D.S. Hesthaven, J. Fluid Mech. 481 (2003) 149.
- [44] P.J. Sengupta, D.S. Hesthaven, Sengupta, Tadmor, Sengupta, Bhowmik, 2001.
- [45] V.C. Pao, R.H. J. Fluid Mech. 38 (1969) 181.
- [46] D.R. Cossin, S.E. Wiggins, M.F. Pao, J. Fluid Mech. 121 (1982) 487.
- [47] B. Hesthaven, C.W.H. D. Sengupta, J. Wiggins, F.T.M. N. Sengupta, H. Foy, B. E. Sengupta, H. Wiggins, R.R. Kang, F. Wiggins, SIAM Review 305 (2004) 1594.
- [48] M.A. Ramesh, P. Resnikoff, SIAM J. Numer. Anal. 81 (11) (1998) 2244.